Controlling network growth

Why controlling network growth?
- Those systems often do not work optimally
- It would be nice to have "social engineering methods" to make them work properly: hindering financial crisis or epidemic outbreaks
- The dynamics of networks, their resilience and also their controllability depend on the network topology

Controlling network growth is difficult:
- Size of state space is growing super-exponentially
- Size of action space is growing with the number of states
- Many real systems are networked systems.

We propose to solve both problem by reducing the control problem to a sampling problem and learning an importance sampler which allows us to solve it approximately.

Stochastic optimal control

Formulation as MDP:
- The state of the network e.g. Adjacency matrix \( X \)
- System transition given a controlling action \( u \): \( P(x_{t+1}^{i}|x_t, u) \)
- System transition given a control \( u \): \( J(x, t) = \min_{u} \{ r(x, t) + J(x', t + 1) \}_{P(x'|x,t,u)} \)

Approximate control problem

We relax the problem:
- Approximate MDP by a linear solvable MDP which can be formulated as a sampling problem

Solving the sampling problem

- Proposal distribution (with functional form of true optimal distribution):
  \( \tilde{u}_i(x_i, t) \propto p(x_i) \exp \left(-\frac{J_{KL}(x_i, t)}{\lambda} \right) \)
  \( \tilde{x}_{KL}(x_i, t) = \frac{1}{Z(t)} \sum_{i=1}^{N} \tilde{u}_i(x_i, t) \phi(x_i) \)

References